Oscillatory Behavior of a Two-phase Natural-circulation Loop

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A natural-circulation loop with water as the circulating fluid was studied for a range of operation covering two-phase flow. The work reported in this paper is concerned with the periodic oscillations of the flow rate and fluid temperature. The oscillations occurred even with constant heat input and constant cooling-water properties for the heat exchanger. The analytical approach includes a theoretical analysis of an open-ended system and numerical solutions obtained with an analogue computer for a simplified loop system. Also presented are the equations of motion, continuity, and energy, which were developed for a transient two-phase flow model for adaptation to more detailed numerical evaluations.

A program of study of the transient operation of natural-circulation loops has been underway at the University of Minnesota (*1*) and this paper is concerned with the oscillatory behavior of a two-phase natural-circulation loop. These studies are of interest for the emergency cooling of nuclear reactors and in the design of boiling-water reactors. The literature survey pertaining to the transient operation of a natural-circulation loop is given by Alstad, Isbin, Amundson, and Silvers (*1*), and a survey of two-phase flow literature is given by Isbin, Moen, and Mosher (*2*).

EXPERIMENTAL LOOP

Figure 1 is a schematic diagram of the natural-convection loop which was studied. The loop was constructed primarily of 16-gauge, 1-in. O.D. (0.872-in. I.D.) hard-drawn brass tubing. The major features of the loop are described in reference *1*. During a natural-circulation run the flowrators were by-passed and only the electromagnetic flow meter was used. Normally the surge tank, E_1, was not used for two-phase natural-circulation runs. For those runs in which the pressure at one point in the loop was held constant, the gate valve between the surge tank E_2 and the loop was opened; for the constant-volume run, the gate valve was closed. A heater and pump were installed to maintain the cooling-water supply at 5 gal./min. and at 130°F.

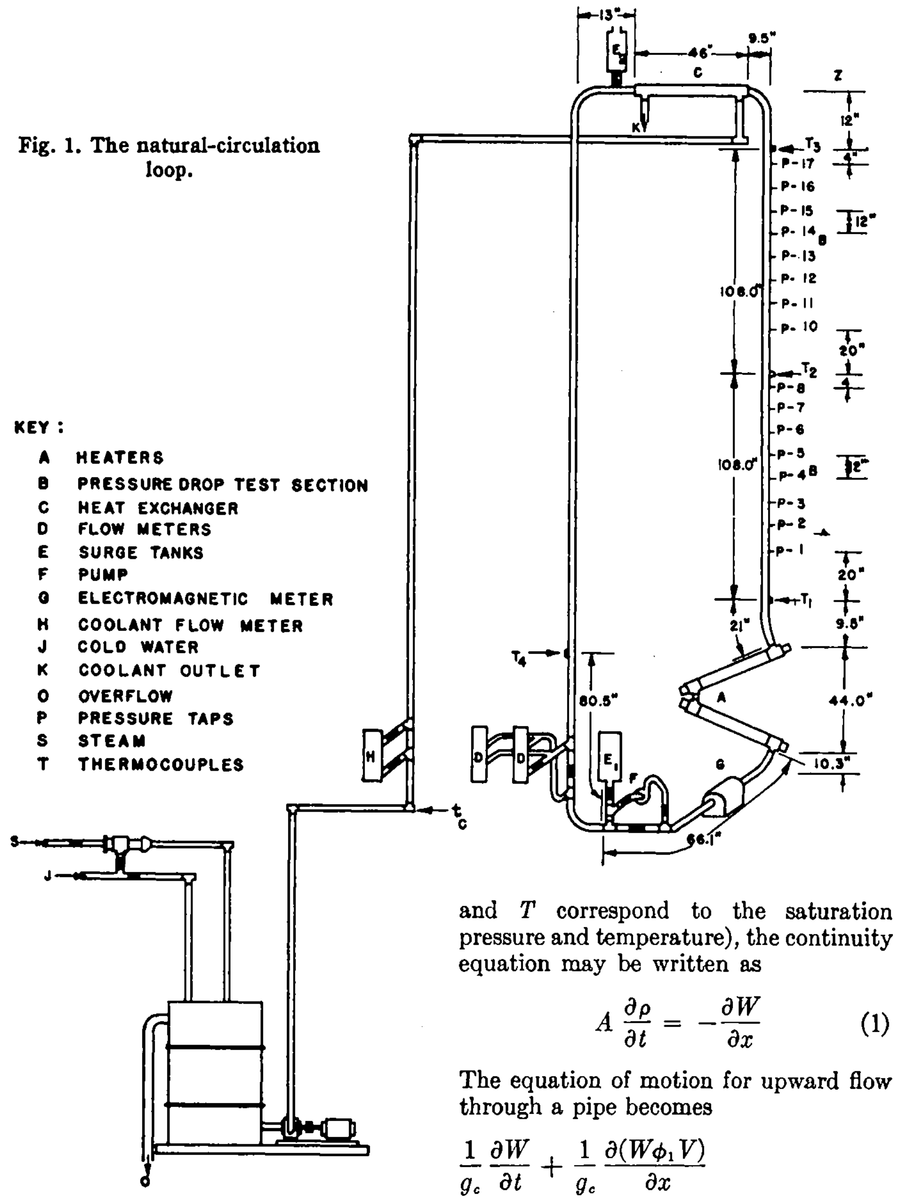

Fig. 1. The natural-circulation loop.

THEORETICAL ANALYSIS

General Equations

The continuity equation, the equation of motion, and the energy equation for a viscous fluid flowing in a region of general geometry were formulated for a phase having continuous properties. A similar set of equations was derived for flow across a surface of discontinuity. The combination of these two sets of equations gave the equations for the two-phase flow. If these equations are applied to flow through a pipe of uniform cross section in which the steam and water phases are assumed to be in equilibrium (that is, p and T correspond to the saturation pressure and temperature), the continuity equation may be written as

$$A \frac{\partial \rho}{\partial t} = -\frac{\partial W}{\partial x} \qquad (1)$$

The equation of motion for upward flow through a pipe becomes

$$\frac{1}{g_c}\frac{\partial W}{\partial t} + \frac{1}{g_c}\frac{\partial (W\phi_1 V)}{\partial x} = -A\frac{\partial p}{\partial x} - A\frac{g}{g_c}\rho z' - AF_f \qquad (2)$$

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Finally the energy equation is

$$A\frac{\partial(\rho\epsilon)}{\partial t} + \frac{\partial(W\phi_2\epsilon)}{\partial x} - \frac{A}{J}\frac{\partial p}{\partial t} \tag{3}$$
$$= Q + \frac{\partial}{\partial x}\left[(k_w A_w + k_s A_s)\frac{\partial T}{\partial x}\right]$$

The terms ϕ_1 and ϕ_2 represent the ratio of the true rate of transport to the rate of transport of the mean flow for momentum and energy respectively. If both phases have the same linear velocity, ϕ_1 and ϕ_2 are unity.

The three functions ϕ_1, ϕ_2, and F_f were determined experimentally from steady state data. It was found that ϕ_1 and ϕ_2 could be correlated as functions of ρ alone and that

$$F_f = a(\rho)W^{1.79} \tag{4}$$

where $a(\rho)$ is a function of ρ. Figures 2, 3, and 4 illustrate the variation of ϕ_1, ϕ_2, and $a(\rho)$ with ρ. Equations (1) through (4) have been applied to a natural-circulation loop in the form of finite-difference equations. The discussion of these equations is not included in this paper as the numerical calculations by means of the S.E.A.C. (National Bureau of Standard's digital computer, Standard's Eastern Automatic Computer) have not been successfully completed. The pressure-drop data will be reported separately.

STABILITY ANALYSES

An insight into the factors which determine the stability of a natural-convection system is gained through the analysis of an open-ended loop, such as the one shown in Figure 5. The fluid entering the heater always has the constant temperature T_0, and the velocity of the stream is fixed by the density difference between the fluid in the hot and cold legs. For any constant heat input, one may define a state of equilibrium in which the difference in weight of the two legs is just equal to the frictional resistance to flow. Under certain conditions this system may be unstable; that is, a small deviation from the equilibrium temperature distribution or the equilibrium flow may be propagated in space or time with increasing amplitude. The problem is to determine those conditions under which a flow perturbation of the form $\epsilon_V \sin \omega t$ can cause a sufficiently large change in the external force acting on the system to sustain or increase the flow perturbation. The problem has been treated analytically for a one-phase fluid (*3*), and the following conclusions were obtained:

1. If an oscillatory flow rate is to be possible, the force cannot be generated in the heater; it must be generated in the vertical riser.
2. The product of the coefficient of expansion of the fluid and the vertical height of the riser must exceed a certain value (defined by an analytical expression) if the flow perturbation is to be sustained.
3. The period of oscillation will be approximately equal to the residence time of the fluid in the heater and the vertical riser.

In order to predict the period for a closed loop, a problem was solved on a Reeves Electronic Analog Computer. The number of nonlinear terms was limited to sixteen, and the model used was necessarily a simple one. The computer could handle only ten subdivisions, with the driving force expressed as a linear function of all ten enthalpies and the frictional force as a quadratic function of the velocity. The number of available summing amplifiers limited the problem to the case in which boiling occurred only at the top subdivision of the vertical riser. The loop equations used were derived as noted in reference *1*.

The energy equation has been simplified in the following manner:

1. $\epsilon \simeq H$. The kinetic- and potential-energy terms were always less than 0.5% of the corresponding enthalpy terms.

2. $\frac{\partial}{\partial x}\left[(k_w A_w + k_s A_s)\frac{\partial T}{\partial x}\right]$

omitted. The axial conduction term was always less than 0.001% of the term $\partial/\partial x\ (W\phi_2 H)$.

3. $\frac{A}{J}\frac{\partial p}{\partial t}$

omitted. An error of less than 0.004% is caused by the omission of this term.

4. $\phi_2 = 1$. The analysis was restricted to a simple model. With these conditions, Equation (3) reduces to

$$\frac{\partial H}{\partial t} = -\frac{V}{A}\frac{\partial H}{\partial x} + \frac{Q}{A\rho} \tag{5}$$

The integration of the equation of motion [Equation (2)] around the loop leads to

$$\frac{dV}{dt} = \frac{-\oint \frac{g}{g_c}\rho z'\,dx - \oint F_f\,dx}{\oint \frac{\rho}{A}\frac{dx}{g_c}} \tag{6}$$

with the added restriction that $\phi_1 = 1$.

Equations (5) and (6) were applied as

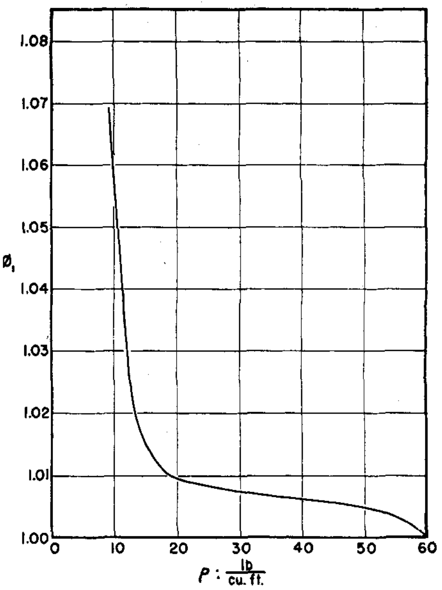

Fig. 2. The momentum-transport coefficient, ϕ_1.

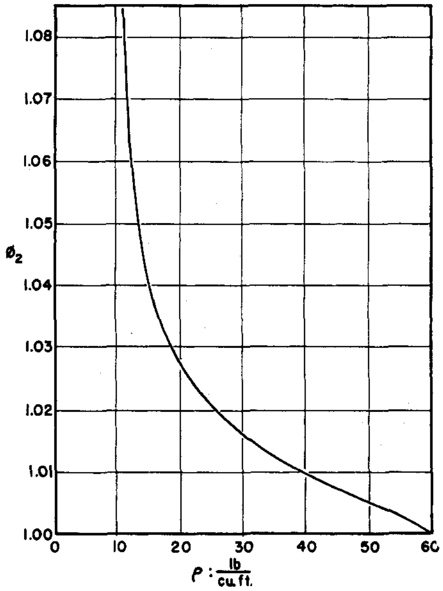

Fig. 3. The energy-transport coefficient, ϕ_2.

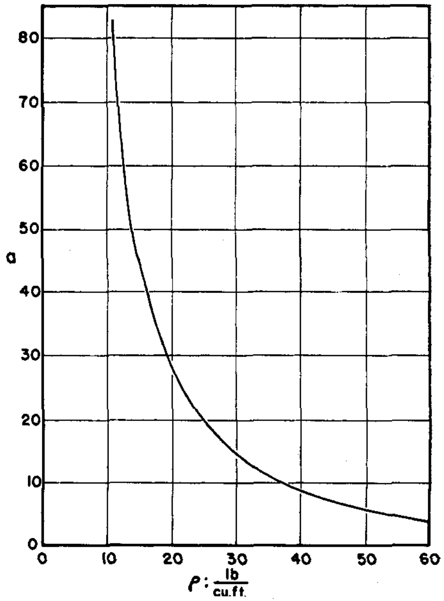

Fig. 4. The friction coefficient, $a(\rho)$.

differential-difference equations and the loop was subdivided as shown in Figure 6. Table 1 summarizes the equations and data used for the analogue-computer problem.

The density at point n was written as

$$\rho_n = \rho_{n_0} + \alpha_n(H_n - H_{n_0}) \qquad (7)$$

and the mean density of the nth subinterval was set equal to the average of the densities at n and $n-1$. A stable solution was found if the coefficient of expansion of water was used for all values of α_n. An oscillatory solution, Figure 7, was obtained if vaporization to a few per cent quality was assumed in the top section of the vertical riser.

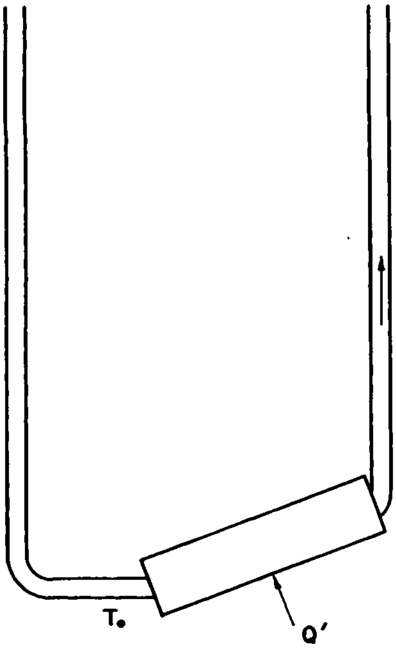

Fig. 5. The stability model.

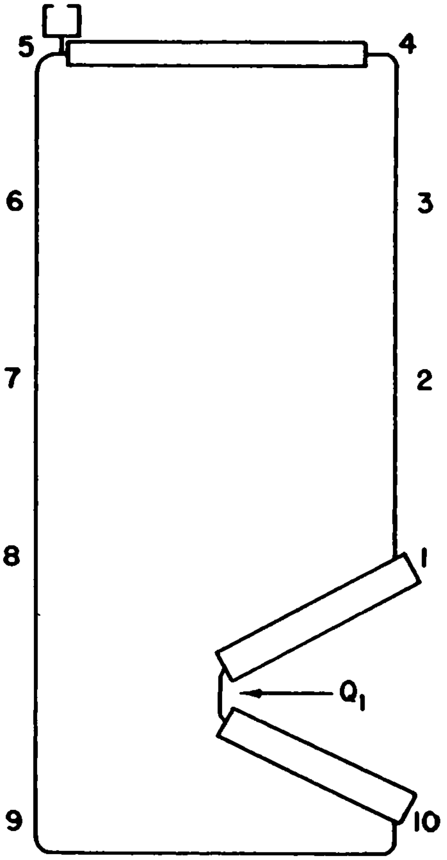

Fig. 6. Subdivisions for the analogue-computer problem.

TABLE 1. DATA FOR THE ANALOGUE COMPUTER

n	$(A\Delta x)_n$, cu. ft.	H_{n_0}, B.t.u./lb.	ρ_{n_0}, lb./cu. ft.	α_n, lb./(cu. ft.)(B.t.u./lb.)	z_n, ft.
1	0.23000	182	59.781	−0.024	3.7084
2	0.02387	182	59.781	−0.024	6.5278
3	0.02387	182	59.781	−0.024	6.5278
4	0.02387	182	37.500	−9.500	6.5278
5	0.01749	153	60.477	−0.024	0
6	0.02387	153	60.477	−0.024	−6.5278
7	0.02387	153	60.477	−0.024	−6.5278
8	0.02387	153	60.477	−0.024	−6.5278
9	0.01356	153	60.477	−0.024	−3.7084
10	0.04504	153	60.477	−0.024	0

Energy equation:

$$\frac{dH_n}{dt} = -\frac{V}{(A\ \Delta x)_n}(H_n - H_{n-1}) + \frac{Q_n}{A_n\bar{\rho}_n}$$

Flow equation:

$$\frac{dV}{dt} = \frac{-\frac{g}{g_c}\sum \bar{\rho} z_n'\ \Delta x - F}{\frac{1}{g_c}\sum \frac{\bar{\rho}_n\ \Delta x_n}{A_n}}$$

$$\bar{\rho}_n = \tfrac{1}{2}(\rho_n + \rho_{n-1})$$

$Q_1 = 3.16$ B.t.u./sec.; $Q_2 = Q_3 = Q_4 = 0$

$Q_5 = -25.0V[((H_5 + H_4)/2) - H_c]$ for the cooler. The heat transfer coefficient was taken to be a linear function of the flow rate, and the temperature driving force was approximated by the enthalpy differences.

A Taylor series was used for the approximation of the frictional force:

$$F \simeq F_0 + \frac{1.8F_0}{V_0}(V - V_0) + \frac{0.72F_0}{V_0^2}(V - V_0)^2$$

RESULTS

Two steady-equilibrium modes of operation are possible when the pressure at one point in the loop is held constant (surge tank E_2 open to the atmosphere). For a very low heat input the water temperature in the riser never exceeds the boiling point and a state of stable equilibrium may be defined. For a very high heat input, the entire riser contains both steam and water, and a maximum flow rate is obtained. Oscillatory modes of operation result for the intermediate heat inputs. The manner in which the period and amplitude depend on the heat input is illustrated by Figures 8 to 12. The period and amplitude of the oscillations are determined by the mean temperature level of the fluid in the vertical riser.

The period is inversely proportional to the mean velocity provided that some steam is in the riser at all times. When the system does not contain some steam during most of the cycle, the period is considerably longer than that predicted by the extrapolation of the higher flow-rate periods.

In the analogue-computer problem $Q_1 = 3.16$ B.t.u./sec. and the period is 149 sec., which is about 21 sec. less than the experimentally observed value for the lowest flow rate. The computed period would be expected to be less than the observed value since the density function used in the top subinterval of the riser was a two-phase density function which implied that the system contained some steam during all the cycle. Further the density function used for the boiling subinterval was linear, and it was not possible to exclude densities greater than the density of saturated water. A special nonlinear element would be required to generate a density function with the correct properties. As a result, the computed mean flow rate is less than that which could actually exist; however, the flow-rate curve and enthalpy curves have essentially the same shape as the experimentally determined ones. A larger computer would be required for solving the stability problem at the higher heat fluxes.

CONCLUSIONS

A natural-circulation loop can be made unstable in the sense that a small displacement from the equilibrium state leads to undamped oscillations. Stable operations result when the fluid temperature in the riser is restricted to values less than the boiling point and when the heat input reaches a value such that the frictional force changes more rapidly than the driving force. The theoretical treatment of an open-ended natural-circulation system and the solution of a simple problem by means of an analogue computer have lent support to the general conclusions of the stability analyses.

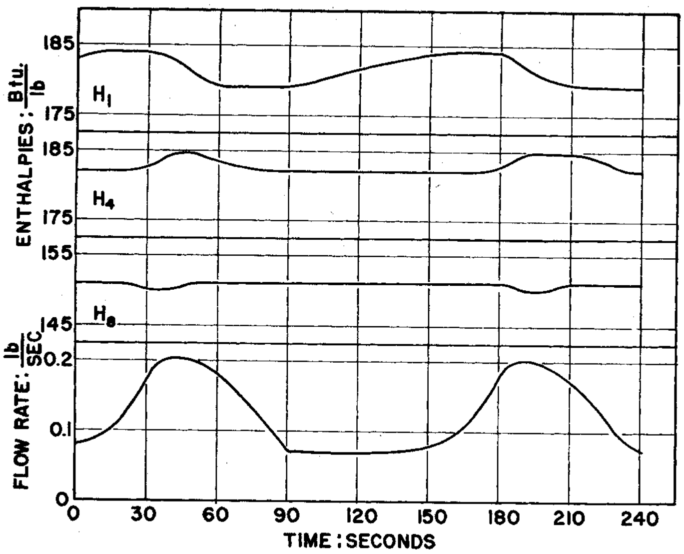

Fig. 7. Solution of the analogue-computer problem.

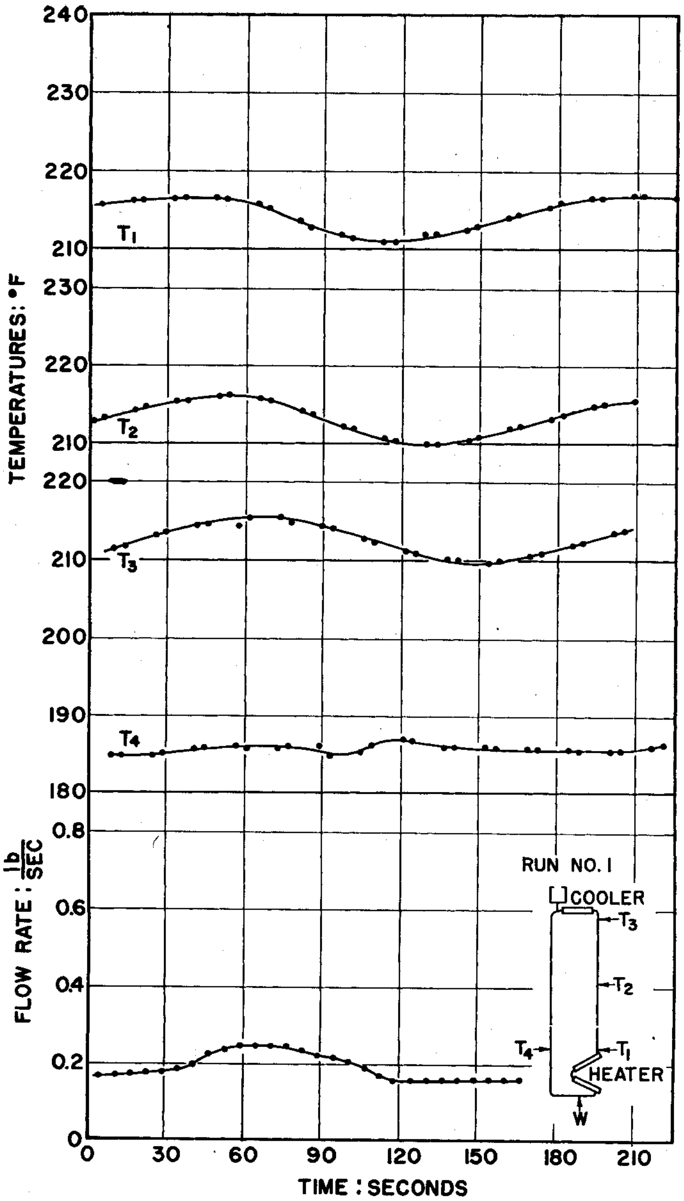

Fig. 8. Flow and temperature oscillations with a low heat input (Q = 5.84 B.t.u./sec.; surge tank open to atmosphere).

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NOTATION

$a(\rho)$ = an empirically defined function for Equation (4)

A = cross-sectional area for flow; A_w = cross-sectional area for water-phase flow; A_s = cross-sectional area for steam-phase flow

g = local acceleration of gravity

g_c = conversion factor in Newton's law of motion

F_f = frictional force of the wall per unit length; F = loop resistance at flow rate V; F_0 = loop resistance at flow rate V_0

H = specific enthalpy of fluid; H_c = specific enthalpy of the cooling water

J = mechanical equivalent of heat

k_w = thermal conductivity of water; k_s = thermal conductivity of steam

p = fluid pressure

Q = linear rate of heat input to the fluid; Q_n = heat input per subdivision

t = time

T = fluid temperature

u = mean linear velocity; u_w = mean linear velocity of water phase; u_s = mean linear velocity of the steam phase

$$u = \frac{A_w \rho_w u_w + A_s \rho_s u_s}{A \rho} = \frac{W}{A \rho}$$

V = mean volumetric flow rate; V_0 = initial or equilibrium volumetric flow rate in loop; $\epsilon_V \sin \omega t$ = perturbation in the flow rate

W = total mass flow rate

x = distance along stream line

z = vertical height; z' = slope of pipe at x

α = coefficient of expansion used in Equation (7); the enthalpy difference was used to approximate the temperature difference

ϵ = specific fluid energy =

$$H + \frac{u^2}{2g_c J} + \frac{g}{g_c}\frac{z}{J}$$

with subscripts s and w designating steam and water phases; mean specific fluid energy

$$\epsilon = \frac{A_w \rho_w \epsilon_w + A_s \rho_s \epsilon_s}{A \rho}$$

ρ = mean fluid density; ρ_w = density of water phase; ρ_s = density of steam phase;

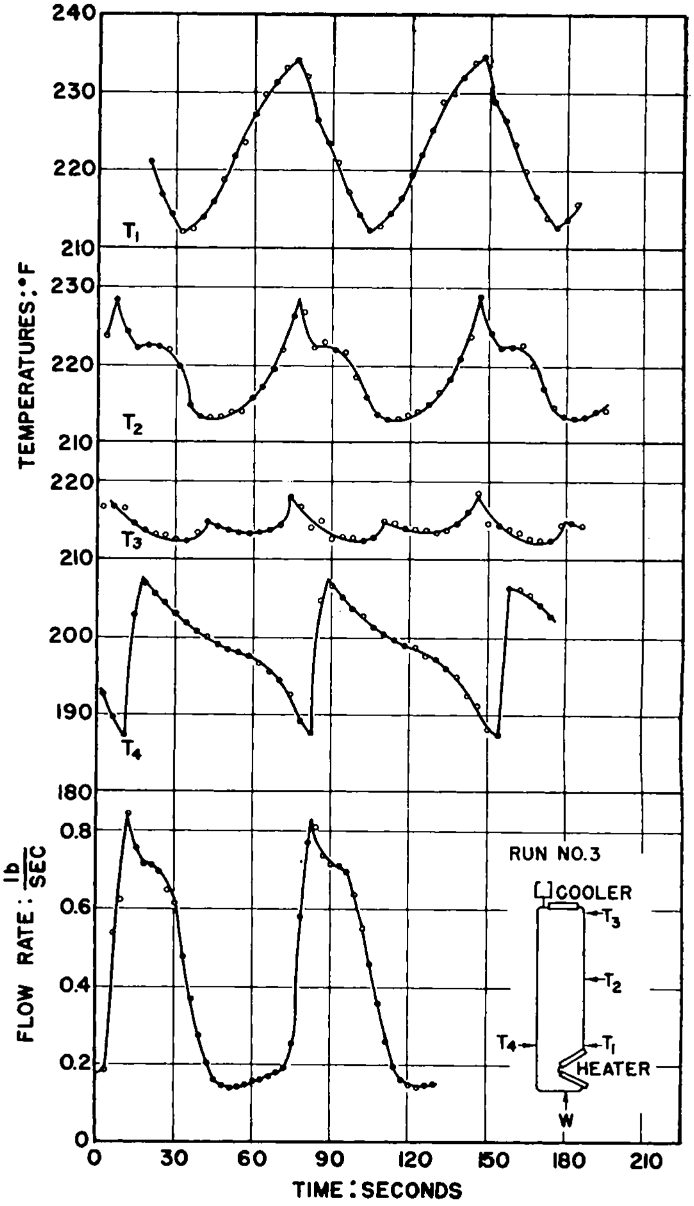

Fig. 10. Flow and temperature oscillations (Q = 9.44 B.t.u./sec.; surge tank open to atmosphere)

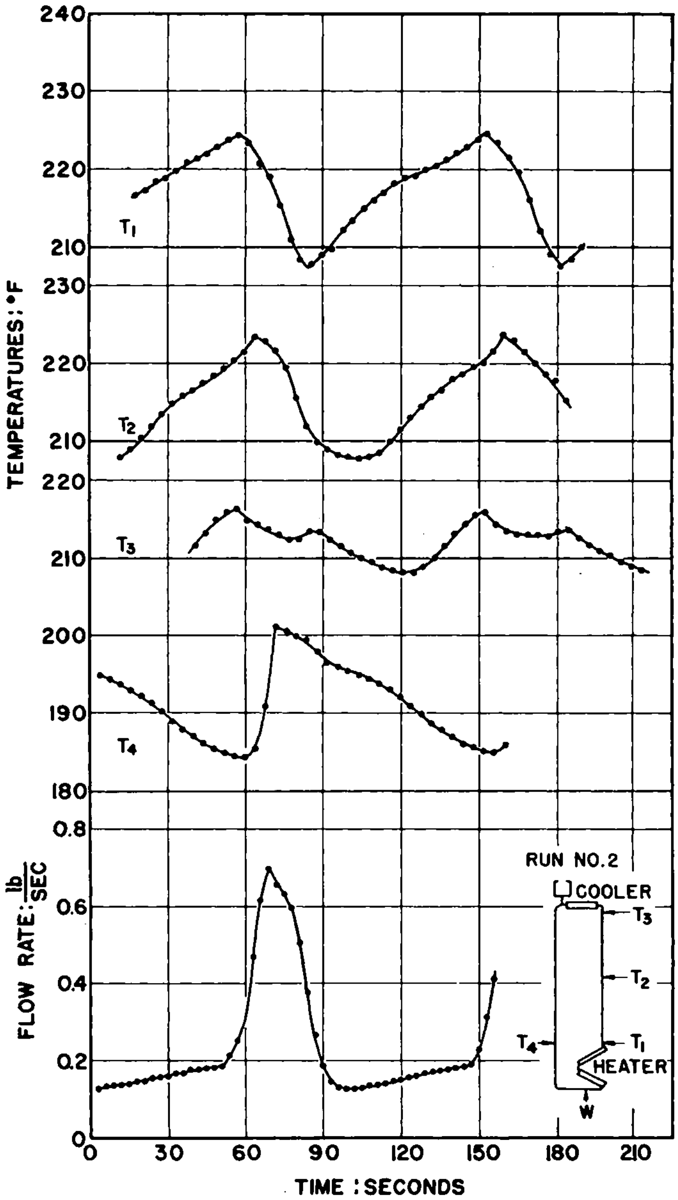

Fig. 9. Flow and temperature oscillations (Q = 7.75 B.t.u./sec.; surge tank open to atmosphere).

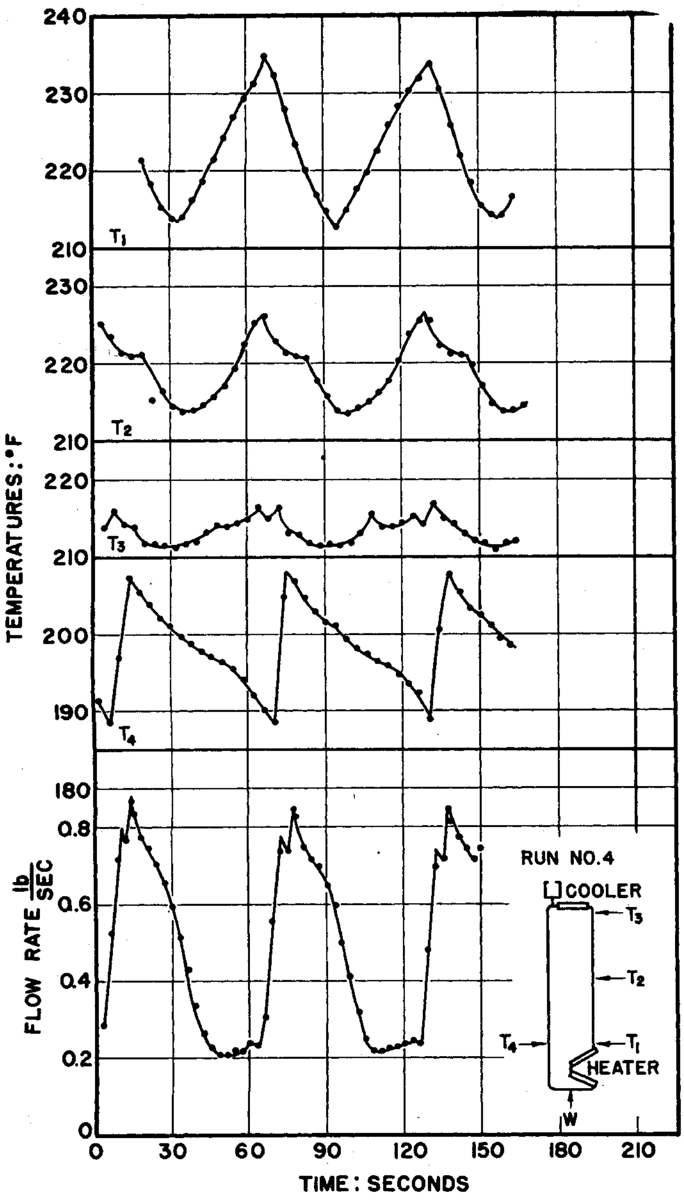

Fig. 11. Flow and temperature oscillations (Q = 11.80 B.t.u./sec.; surge tank open to atmosphere)

Fig. 12. Flow and temperature oscillations (Q = 12.70 B.t.u./sec.; surge tank open to atmosphere)

$$\rho = \frac{1}{A} \cdot (A_w \rho_w + A_s \rho_s)$$

$$\phi_1 = \frac{A_w \rho_w u_w^2 + A_s \rho_s u_s^2}{W u}$$

$$\phi_2 = \frac{A_w \rho_w u_w \epsilon_w + A_s \rho_s u_s \epsilon_s}{W \epsilon}$$

Subscripts

n = subdivision number
s = steam phase
w = liquid-water phase
0 = initial conditions

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